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TECHNICAL NOTE

FAILURE ANALYSIS OF MASONRY SHEAR WALLS USING DISCONTINUOUS DEFORMATION ANALYSIS

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ABSTRACT

This paper presents a method for failure analysis and studying post failure behavior of masonry shear wall. Masonry structures are blocky systems that have great importance in civil engineering. Masonry structures such as stone and brick walls, arches and vaults in tunnel supports and bridges, constitute a great percent of existing structures especially some with historical value all over the world. It is very important that a method can analytically model masonry structures and their behavior under vertical and horizontal loads in order to determine their short and long-term stability. Masonry structures generally consist of intact units (masonry blocks, stone, brick, clay units) separated by discontinuities (mortar joints). Laboratory and field observations have showed that these discontinuities have a strong and non-linear effect and dominate the deformation and strength behavior of masonry structures. The occurrence and propagation of the brittle cracking and sliding also have a great effect in behavior of masonry structures. This modified DDA method can capture the behavior and can handle the problem adequately. For considering the effect of cracking and fracturing in blocks some modifications to the original method have been developed and implemented in the original DDA code. Finally with some simple examples capability of this new method has been demonstrated.

Keywords: DDA, masonry shear wall, fracture, discontinuous deformation, dynamic analysis

1. INTRODUCTION

The Discontinuous Deformation Analysis (DDA) Method is a recently developed technique that is a member of the family of DEM methods. In 1988, Shi published his Ph.D thesis, *"Discontinuous Deformation Analysis: A New Numerical Model for the Statics and Dynamics of Block Systems"*. In the DDA method the formulation of the block is very similar to the definition of a finite element mesh. A finite element type of problem is solved in which all elements are physically isolated blocks bounded by pre-existing discontinuities. The elements or blocks used by the DDA method can be of any convex or concave shape whereas the FEM uses only elements with predetermined topologies. When blocks are in contact, Coulomb's law applies to the contact interfaces, and the simultaneous equilibrium equations are selected and solved at each loading or time increment.

The large displacements and deformations are the accumulation of small displacements and deformations at each time step. Within each time step, the displacements of all points are small, hence the displacements can be reasonably represented by first (higher) order approximations.

In the DDA method, individual blocks form a system of blocks through contacts among blocks and displacement constraints on single blocks. Shi showed that the simultaneous equilibrium equations can be expressed as KD=F where K is stiffness matrix and D and F are displacement and force matrices, respectively. In total, the number of displacement unknowns is the sum of the degrees of freedom of all the blocks. It is noted that this system of equations is similar in form to that in finite element problems.

The solution to the system of equations is constrained by a system of inequalities associated with kinematics (e.g. no penetration no tension between blocks) and Coulomb friction for sliding along block interfaces. The system of equations is solved for the displacement variables.

Both static and dynamic analyses can be conducted with the DDA method.

2. BLOCK DEFORMATION AND DISPLACEMENT

The large displacements are summation of small displacement in steps. Within each time step, the displacements of all points are small and the displacement functions (as shape function in FEM) can be simplified.

By using first order displacement approximation, the DDA method assumes that each block has constant strains and stresses throughout. The displacements (u,υ) at any point (x,y) in a block, i, can be related in two dimensions to six displacement variables

$$\mathbf{D}=\left(d_{1i},d_{2i},d_{3i},d_{4i},d_{5i},d_{6i}\right)^T=\left(u_0,v_0,r_0,\varepsilon_x,\varepsilon_y,\gamma_{xy}\right)^T$$

Where (u_0,υ_0) is the rigid body translation at a specific point in the block, r_0 is the rotation angle of the block and ε_x, ε_y and γ_{xy} are the normal and shear strains in the block.

In the two dimensional formulation of the DDA, the center of rotation with coordinates (x_0,y_0) coincides with the block centroid with coordinates (x_c,y_c).

2.1 Equilibrium Equations

In the DDA method, individual blocks form a system of blocks through contacts among blocks and displacement constraints on single blocks. Assuming that n blocks are used in the block system, Shi showed that simultaneous equilibrium equations can be written in matrix form.

$$\begin{bmatrix} K_{11} & K_{12} & \cdot & \cdot & K_{1n} \\ K_{21} & K_{22} & \cdot & \cdot & K_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ K_{n1} & K_{n2} & \cdot & \cdot & K_{nn} \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \\ \cdot \\ \cdot \\ D_n \end{bmatrix}=\begin{bmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ F_n \end{bmatrix}$$

Where each coefficient K_{ij} is defined by the contacts between blocks i and j. Since each block has six degree of freedom defined by components of D_i, each K_{ij} is itself a 6×6 sub matrix. The system of equations can be expressed in compact form (similar to finite element) as

$$\mathbf{KD} = \mathbf{F}$$

Where **K** is a $6n \times 6n$ stiffness matrix and **D** and **F** are $6n \times 1$ displacement and force matrices.

The solution to the system of equation is constrained by a system of inequalities associated with block kinematics.

The simultaneous equations were derived by shi minimizing the total potential energy Π of the block system. The total potential energy is the summation over all the potential energy sources:

1. The strain potential energy Π_e produces stiffness matrix,
2. The potential energy Π_σ of initial stresses produces the initial stress matrix,
3. The potential energy Π_p of point load produces the point load matrix,
4. The potential energy Π_w of bobby load produces the body load matrix,
5. The potential energy Π_i of inertia produces mass matrix,
6. The strain potential energy Π_s of contact (normal and shear) springs produces contact matrix,
7. The potential energy Π_f of friction force.

Similar to FEM method by minimizing the total potential energy all the block matrices would be produced.

$$\frac{\partial^2 \Pi}{\partial d_{ir}\, \partial d_{js}},\ r, s = 1.....6$$

For FEM the integration domains of the block matrices are whole elements with standard boundaries, but for DDA method, the integration domains of the block matrices are blocks. In the next section the procedure for derivation of the formulation of the elastic matrix or minimization of strain energy is discussed.

2.2 Elastic Matrix for DDA

The strain energy done by the elastic stresses of block i is:

$$\Pi_e = \iint \frac{1}{2}\varepsilon\sigma dxdy = \iint_A \frac{1}{2}\left(\varepsilon_x \sigma_x + \varepsilon_y \sigma_y + \gamma_{xy} \tau_{xy}\right) dxdy$$

Where the integration is over the block area and for condition of plane stress

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-v^2}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [E]\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

E, υ are the Young's modulus and Poisson's ratio, respectively.

By substituting the stress-strain relationship in the strain energy formulation, we have

$$\Pi_e = \iint_A \frac{1}{2}\begin{pmatrix}\varepsilon_x & \varepsilon_y & \gamma_{xy}\end{pmatrix}\begin{bmatrix}\sigma_x \\ \sigma_y \\ \tau_{xy}\end{bmatrix} dxdy$$

$$= \iint_A \frac{1}{2}\begin{pmatrix}\varepsilon_x & \varepsilon_y & \gamma_{xy}\end{pmatrix}[\mathbf{E}]\begin{Bmatrix}\varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}\end{Bmatrix} dxdy$$

$$= \frac{1}{2}\iint_A [D_i]^T [E][D_i] dxdy$$

$$= \frac{1}{2}[D_i]^T \left[\iint_A [E] dxdy\right][D_i]$$

$$= \frac{S}{2}[D_i]^T [E][D_i]$$

where S is the area of the block. By differentiating we have

$$K_{rs} = \frac{\partial^2 \Pi_e}{\partial d_{ri} \partial d_{sj}} = \frac{S}{2}\frac{\partial^2}{\partial d_{ri} \partial d_{sj}}[D_i]^T [E][D_i]$$

which K_{rs} forms a 6×6 sub matrix.

$$S[E] \rightarrow [K_{ii}]$$

Which is added to the global matrix.

We can continue this procedure for all sort of loading such as point loading, line loading, volume loading or body force, bolt connection, ... as Shi described in his work (Shi [6]). or for time history force or earthquake loading in other researchers work such as Naderi [4].

3. BLOCK KINEMATICS

Despite the similarities between DDA and FEM in derivation of the formulation of element or block matrices, compatibility and continuity equations can not be satisfied in DDA as easily as FEM.

In order to extend the formulation to a rock mass, it is necessary to connect the individual blocks into a block system. For block system movements, no tension and no penetration inequalities can be used and these equations must be imposed on the global equilibrium equation.

In DDA Shi developed an entrance theory for satisfying these conditions. The entrance theory is in fact a system of inequalities associated with block kinematics for no penetration and no tension between blocks and Coulomb friction for sliding along block interfaces.

3.1Contacts and Interpenetrations

There are different kinds of contacts between blocks: angle to angle, angle to edge and edge to edge. Edge to edge contacts can be transferred into two angle to edge contacts, Figure 1.

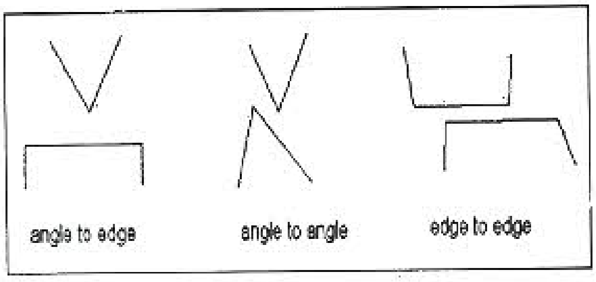

Figure 1 Different Kinds of contacts

Two rules must be obeyed when there is contact:

1. No interpereation occurs between the two sides.
2. No tension force exists between the two sides.

3.2 The Sub matrix of a Contact

Consider a spring between point p_1 and reference line p_2p_3 in Figure 2.

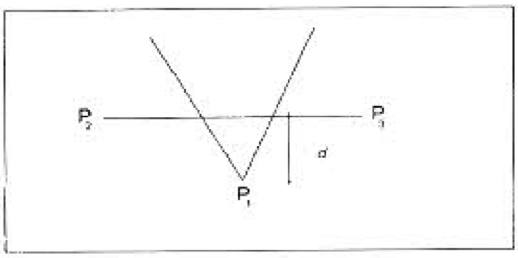

Figure 2 the distance between blocks

Denote (x_i,y_i) and (u_i,v_i) as the coordinates and displacement increments at points $p_i(i=1,2,3)$ respectively.. After solving the equilibrium equation the displacement increments can be calculated, to push point p_1 back from interpenetration line p_2,p_3, the perpendicular distance d between point $p_1(x_1,y_1)$ and the reference edge p_2p_3 can be expressed as

$$d = \frac{\Delta}{l} = \frac{1}{l}\begin{vmatrix} 1 & x_1 + u_1 & y_1 + v_1 \\ 1 & x_2 + u_2 & y_2 + v_2 \\ 1 & x_3 + u_3 & y_3 + v_3 \end{vmatrix}$$

Where l is the length of p_2p_3 and can be calculated by

$$l = \sqrt{(x_2 + u_2 - x_3 - u_3)^2 + (y_2 + v_2 - y_3 - v_3)^2}$$

And Δ is such that

$$\Delta = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} + \begin{vmatrix} 1 & u_1 & y_1 \\ 1 & u_2 & y_2 \\ 1 & u_3 & y_3 \end{vmatrix} + \begin{vmatrix} 1 & x_1 & v_1 \\ 1 & x_2 & v_2 \\ 1 & x_3 & v_3 \end{vmatrix} + \begin{vmatrix} 1 & u_1 & v_1 \\ 1 & u_2 & v_2 \\ 1 & u_3 & v_3 \end{vmatrix}$$

Denote the stiffness of the spring as p and the penetration distance as d, the strain energy of the contact spring is

$$\Pi_k = \frac{p}{2} d^2$$

Where p is a very large positive number and is the stiffness of the spring.

The value of p is normally from 10E to 1000E, to satisfy the displacement of the spring is 0.001 to 0.0001 times the total displacement. If p is large enough, the computation result will be independent of the choices of p, then the potential energy will be

$$\Pi_k = \frac{p}{2}\left(\sum_{r=1}^{6} e_r d_{ri} + \sum_{r=1}^{6} g_r d_{rj} + \frac{S_o}{t} \right)^2$$

Minimizing Π_k, four 6 × 6 sub matrices and two 6 × sub matrices are obtained and they would be added to

$$\left[K_{ii}\right]\left[K_{ij}\right]\left[K_{ji}\right]\left[K_{jj}\right]\left[F_i\right]\left[F_j\right]$$, respectively.

The derivatives of Π_k

$$K_{rs} = \frac{\partial^2 \Pi_k}{\partial d_{ri} \partial d_{sj}} = p e_r e_s \qquad r, s = 1,\dots,6$$

form a sub matrix

$$p \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{bmatrix} \begin{pmatrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{pmatrix} \rightarrow \left[K_{ii}\right]$$

which would be added to the global equation.

By the same procedure, we would obtain all other sub matrices.

3.3 Block Fracturing

In jointed rock mass analysis, the most important factor is the geometry of the rock mass or blocks. But when we consider block fracturing a new important factor will emerge, this important factor is the shear strength of the potential failure surface or fracture criterion. For failure criterion there has been many studies to model this complex behavior of rocks. The first and the most popular failure criteria in soil and rock mechanics is Mohair-Coulomb criterion.

In this paper this criterion has been used in order to determining block fracturing.

4. ILLUSTARTING EXAMPLES

4.1 Masonry Shear Wall Analysis I

Masonry structures are blocky systems that have great importance in civil engineering. Masonry structures such as stone and brick walls, arches and vaults in tunnel supports and bridges, constitute a great percent of existing structures especially some with historical value all over the world. Many of these structures suffer from numerous deficiencies related to the construction materials, construction techniques and deterioration due to aging. It is very important that a method can analytically model masonry structures and their behavior under vertical and horizontal loads in order to determine their short and long term stability.

Masonry structures generally consist of Intact units (masonry blocks, stone, brick, clay units) separated by discontinuities (mortar joints). Laboratory and field observations have showed that these discontinuities have a strong and non-linear effect and dominate the deformation and strength behavior of masonry structures. The occurrence and propagation of the brittle cracking and sliding also have a great effect in behavior of masonry structures. This modified DDA method can capture the behavior and can handle the problem adequately. Figure 1.1 shows the initial configuration of blocks in a shear wall. The example was designed under the following specifications. The wall consists of 15 blocks with a fixed base and a beam for producing the vertical and horizontal loads. The wall is 1.4m in height and 1.2m in width which sits on a fixed base with an .0.4m × 0.2m on top of it. The blocks are 0.4m × 0.2m in size and the wall consists of layers of stone blocks 2.5 unit per layer. All the blocks have the same material properties with a Young's Modulus of E=8GPa and a Poisson's ratio of υ=0.2. The stone blocks have a unit weight γ=22KN/M^3, a cohesion S_0=3 MPa, a tensile strenght of T_0=2.8 MPa and an internal friction angle of ϕ=30°.

Two kinds of brick joints have been used in the examples. In the first example joints have cohesion of 300 Kpa and a friction angle of 30° and in the second example joints are cohesionless. Shear and tensile fracturing of blocks is allowed depending on the value of principal stresses. In the first example the horizontal load is 3 MN and vertical load is 4 MN. The program was run 200 steps of 0.00002 seconds. Figure 1.2 through Figure 1.5 shows the crack propagation in the wall after 50,100,150 and 200 time steps.

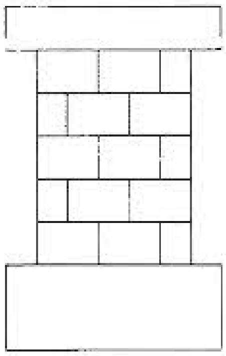

Figure 1.1 Initial Configuration of Blocks in Shear Wall

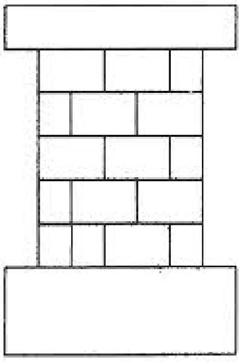

Figure 1.2 Block Displacements after 50 steps of 0.00002 sec

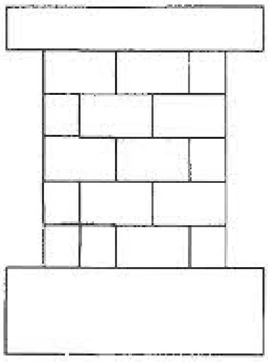

Figure 1.3 Block Displacements after 100 steps of 0.00002 sec

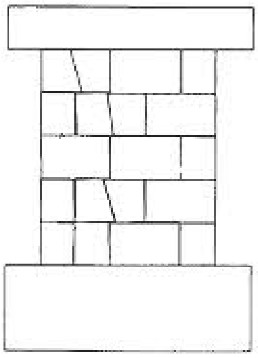

Figure 1.4 Block Displacements after 150 steps of 0.00002 sec

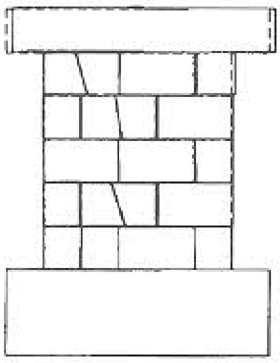

Figure 1.5 Block Displacements after 200 steps of 0.00002 sec

It can be seen that the cracks initiate in the wall near the right toe and the overall failure occurs across the brick units with some effects along the brick joints, the wall would remain initially intact.

4.2 Masonry Shear Wall Analysis II

The second example was run under different material properties and loading condition. The geometry of the wall and bricks is the same as the previous example but the joints are cohesionless and loading condition is 8 MN for horizontal load and 10 MN for vertical load. The program was run 200 steps of 0.00002 seconds. Figure 2.1 through figure 2.6 shows the undeformed and fractured shear wall after 40,80,120,160 and 200 time steps, respectively. It is clear that cracking of the wall starts again at its right toe and cracking propagates upwards. Figure 2.7 shows the final configuration of the wall. It can be seen that the wall deforms and would not remain intact due to sliding along the joints between the blocks. Figure 2.8 shows the failure pattern in the shear wall. It is interesting that the failure pattern is very similar to the observed mode of failure in the field. The purpose of the above examples was to show the capability of DDA in better understanding of the behavior of the masonry structures at the field scale.

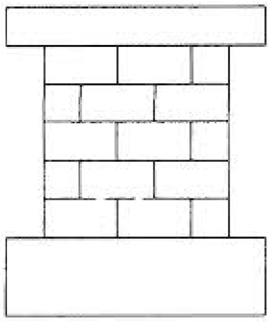

Figure 2.1 Initial Configuration of Blocks in Shear Wall

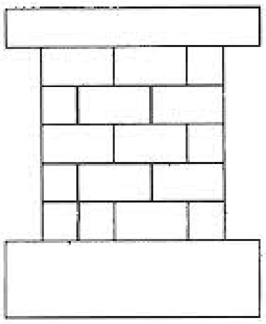

Figure 2.2 Block Displacements after 40 steps of 0.00002 sec

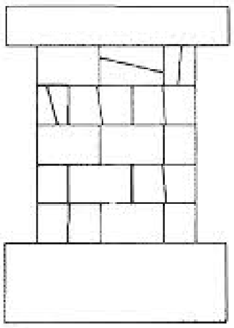

Figure 2.3 Block Displacements after 80 steps of 0.00002 sec

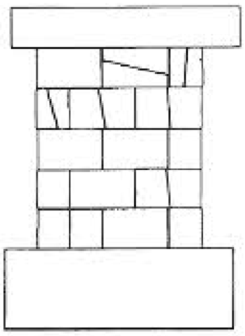

Figure 2.4 Block Displacements after 120 steps of 0.00002 sec

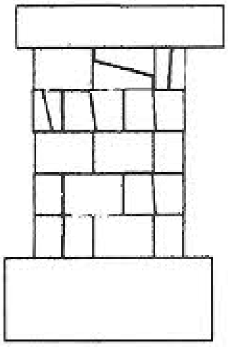

Figure 2.5 Block Displacements after 160 steps of 0.00002 sec

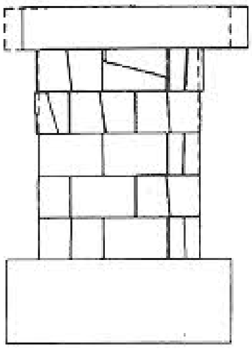

Figure 2.6 Block Displacements after 200 steps of 0.00002 sec

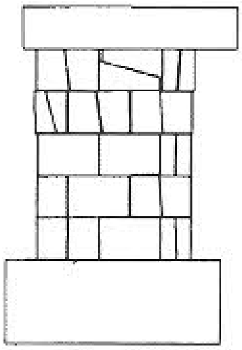

Figure 2.7 Final Configuration of Blocks in Shear Wall

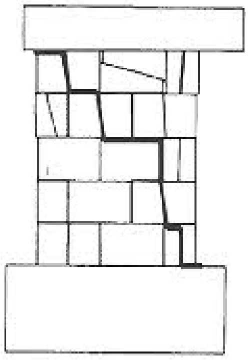

Figure 2.8 Failure Pattern in Shear Wall

The results also show that joints cohesion has a significant effect on the deformation and fracturing of the masonry shear walls.

5. CONCLUSIONS

Over the past ten years, after Shies work many extensions to the original DDA method have been implemented such as: improvement of block contact, determination of stress distributions within the blocks, block fracturing, fracture propagation and .... in this work also it has been shown that in masonry shear wall analysis, the DDA method is very capable and can handle any form of problems. The results also showed a good similarity between observed mode of failure in masonry shear walls and the results which was obtained numerically from the program and indicate the great importance of joint cohesion in overall failure of the wall.

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